**Mathematical induction**

Induction means the inference of general law from particular instances.

OR

Mathematical induction means of proving a theorem by showing that it is true of any particular case, it is true of the next cases in a series and then showing that it is indeed true in one particular case.

**Working procedure of mathematical induction**

Let p (n) be a statement of a theorem for all n ϵ N. Then,

1. Verify the result is true for n = 1.
2. Suppose that the result is true for n = k.
3. Show that the result is true for n = k + 1.

Thus, p (n) is true for all n ϵ N

1. **Prove the statement by using mathematical induction : 2 + 4 + 6 + ……. + 2n = + n, Ɐ n ϵ N**

Proof :- Let p (n) = 2 + 4 + 6 + …….. + 2n = + n.

If n = 1, then, p (1) = 2 = + 1

Since the statement is true for n = 1. Let us assume that the given statement is true for n = k

I.e. p (k) = 2 + 4 + 6 + ……..+ 2k = + k.

Now we have to prove that it is true for n = k + 1.

I.e. p (k +1) is true.

Now, P (k + 1) = 2 + 4 + 6 +……… + 2k + 2 (k +1)

= +k + 2k +2

= ( + 2k + 1) + (k + 1)

= + (k + 1), which is true for n = k + 1.

So, the theorem is true for n = 1. It is also true for n = k. Then, it is concluded true for n = k +1. Hence it is true for all n.

Proved

1. **Prove the statement by mathematical induction for every natural number n the proposition p (n) = + + + ………… + = .**

Proof:- Here the given proposition is p (n) = + + + ………… + = .

If n = 1, then, p (1) = = 1 = .

The proposition is true for n = 1. Let us assume that the proposition is true for n = k

I.e. p (k) = + + + ………… + = .

Now we have to show that the proposition is true for n = k + 1.

I.e. p (k +1) is true.

Now, p (k +1) = + + + ………… + +

= +

= (k + 1) [ + (k + 1)]

= [2 + k +6k +6]

=

=

=

=

= , which is true for n = k + 1.

So, the proposition is true for n = 1. It is also true for n = k. It is concluded true for n = k +1. Hence it is true for all n.

Proved

1. **By using mathematical induction to show that - 1 is divisible by 24.**

Proof : Let p (n) = - 1

If n = 1, then, p (1) = - 1

= -1

= 25 – 1

= 24

So, 24 is divisible by 24. Which implies that the statement is true for n = 1.

Let us assume that the statement is true for n = k

I.e. p (k) = -1 is divisible by 24.

Then, – 1 = 24m for some m

i.e. = 24m +1

Now, we have to show that the statement is true for n = k + 1

i.e. p (k + 1) is divisible by 24.

Now, p (k + 1) = – 1

= . - 1

= ( 24m + 1) . - 1

= 24m . + - 1

= 600m + 24

= 24 (25m + 1) which is divisible by 24.

So, the statement is true for n = 1. It is also true for n = k. It is concluded true for n = k + 1. Hence, the statement is true for all n.

Proved

1. **Let S (n ) = (1 - )(1 - ) …….(1 - ). Prove the statement p (n) : S( n) = by mathematical induction.**

Proof : Let p(n) : S(n) = (1 - )(1 - ) …….(1 - ) =

If n = 1, then, p (1): S (1) = 1 - = =

So, the statement is true for n = 1.

Let us assume that the statement is true for n = k

I.e. p (k): S (k) = (1 - )(1 - ) …….(1 - ) =

Now we have to show that the statement is true for n = k +1

I.e. p (k +1) : S(k +1) is true.

Now, p (k +1): S (k +1) = (1 - )(1 - ) …….(1 - )

= (1 - )

= x

= x

=

= which is true for n = k + 1.

So, the statement is true for n = 1. It is also true for n = k. then, it is concluded true for n = k + 1. Hence, the theorem is true for all n.

Proved

**Binomial Expression**

Binomial expression is an expression consisting of two terms.

Eg. X + 1, x + a, x + y are all binomial expression.

**Factorial notation**

1. n! = n (n – 1) (n – 2)…….3.2.1
2. 5! = 5 .4.3.2.1 = 120

**Permutation**

The arrangement or selection of objects with some order is called permutation.

Eg. Arrangement of three letters A, B, and C are as follows:

1. ABC
2. BCA
3. CAB
4. ACB
5. BAC
6. CAB

So, three letters A, B, C are arranged by 6 ways.

**Formula of permutation**

1. P (n, r) =
2. P (n, r) = (repeated permutation)
3. P (n, r) = (n -1)! (circular permutation)

**Combination**

The selection of objects without order is called combination.

From above example of permutation, three letters A, B, C arranged in 6 ways, but there are same selections of letter A, B, C. So selection of three letters is one ways. Hence, combination of three letter taken from three letters is one ways.

**Formula of combination**

1. C (n, r) =

**Prove that C (n, r) + C (n, r-1) = C (n+1, r), 1≤k≤ n**

Proof: we know that C (n, r) = ………(1)

C (n, r-1) = ........................(2)

Adding (1) and (2), we get

C (n, r) + C (n, r-1) = +

= +

= [ + ]

= [ ]

= [ ]

=

=

= C (n+1, r)

Hence, C (n, r) + C (n, r-1) = C (n+1, r), 1≤k≤ n

Proved.

Binomial Expansion

* = 1
* = a + b = c ( 1, 0)a + c (1, 1)b
* = + 2ab + = c (2, 0) + c (2, 1)ab + c(2, 2)
* = + 3b + 3a+

= c(3, 0) + c(3, 1)b + c(3, 2)a + c(3, 3)

………… …………………………………………………..

………… …………………………………………………..

………… …………………………………………………..

* = c(n, 0) + c( n, 1) b + c(n, 2) + …………+ c(n, r) + ………..+ c(n, n) …….(i)

**Expansion of**

Put b = x in equation (i),

= c (n, 0) + c( n, 1) + c(n, 2) + …………+ c(n, r) + ………..+ c(n, n) …….(ii)

**Expansion of and**

Put a = 1 in (ii ), we get,

1. = c(n, 0) + c( n, 1) + c(n, 2) + ……… …+ c(n, r) + ………..+ c(n, n)

= c(n, 0 ) + c( n, 1) + c(n, 2) + …………+ c(n, r) + ……….. + c(n, n) ……..(iii)

= 1 + n + + …………+ + ………..+ …….. (iv)

Replacing x by – x in (iii), we get,

1. = 1 - n + + ……+ …….. (v)

**Binomials coefficients**

We have,

= c(n, 0) + c( n, 1) + c(n, 2) + ………+ c(n, r) + ……… ..+ c(n, n) ……. (vi)

Put x = 1 I n (vi), we get,

= c(n, 0) + c( n, 1) + c(n, 2) + ………+ c(n, r) + ………..+c(n , n)

**= + + + ………+ = sum of binomial coefficients**

Again put x = -1 in (vi)

= c(n, 0) + c( n, 1)(-1) + c(n, 2) + ………………+ c(n, r) + ………..+ c(n, n)

0 = - + - + ………+

**i.e. + + + ……… = + + + ………**

**Note:**

1. = + + + ………+

+ + ………+ = -

+ + ………+ = – 1

1. + + + ……… = + + + ……… =

**General term of**

1. First term = = c(n, 0)
2. Second term = = c(n, 1)x
3. Third term = = c(n, 2)
4. term = = c(n, r) ( general term )

**Middle term of**

1. If n is even. Then, there is odd no. of term in the expansion. So there is only one middle term.

Middle term = = c(n, )

1. If n is odd. Then, there is even no. of term in the expansion. So there will be two middle terms.

Middle term = = c(n, )

Middle term = = c(n, )

**Pascal’s Triangle**

Pascal’s triangle is given as follows

1 n = 0

1 1 n = 1

1 2 1 n = 2

1 3 3 1 n = 3

1 4 6 4 1 n = 4

Expansion of by using Pascal’s triangle

= +4x + 6 + 4 +

1. Find the expansion of by using binomial theorem.

Solution:

= c(5, 0) + c(5, 1).3 + c(5, 2) + c(5, 3) + c(5, 4) + c(5, 5)

# = 32 +× × + × × 9+ × × 27 + × 2x × 81 + [ c(5, 0) = 1 = c(5, 5) ]

# = + × + × + × + × +

# = +2 + + + + ans

1. Find the term of (.

Solution: Let term is the general term of the expansion of

= c (12, r) ……(i)

Now, for term, r + 1 = 10 r = 10 – 1 r = 9

Using r = 9 on (i), we get

= c (12, 9)

=

=

= × 8 ×

= ans

1. Find the coefficient of in the expansion of

Solution: Let term is the general term of the expansion of

= c (9, r) …… (i)

Now, for coefficient of , we get, 9 – r = 6 or, r = 3

So, = c (9, 3)

=

=

= 84

The coefficient of in the expansion of is 84. Ans

1. Find the general term in the expansion of .

Solution: Let term is the general term of the expansion of

= c (n, r)

= c (n, r)

# = c (n, r)

1. Find the middle term in the expansion of .

Solution: Here n = 6, which is even. So, there is only one middle term in the expansion of .

i.e., =

=

= c (6, 3)

=

= ×27

= 540 ans

1. Find the middle term in the expansion of .

Solution: Here n = 11, which is odd. So, there are two middle terms in the expansion of .

1. = = = = c(11, 5)

= × = = - 462 ans

1. = = = c(11, 6)

= × = = 462 ans

1. Find the term free (or independent) from x in the expansion of .

Solution: Let be the term of free (or independent) of x.

Then, = c (12, r) = c (12, r)

= c (12, r) = c (12, r) ….. (i)

For the term free (or independent) of x,

12 – 3r = 0 3r = 12 r = 4

The term of free (or independent) of x is i.e. term.

Its value is = = c (12, 4) =

= = ans

1. Prove that for n 1, C (2n, n) =

Proof: LHS = c (2n, n)

=

=

=

=

=

= RHS proved